Stability conditions for Dynkin quivers

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1. Background
   - Dynkin quivers and ADE singularities
   - Mirror symmetry
   - Stability conditions

2. Part I : Full σ-exceptional collections
   - Full exceptional collection
   - Results I

3. Part II : Gamma integral structure
   - Invertible polynomials of chain type
   - Results II

A quiver $Q \overset{\text{def}}{\Longleftrightarrow}$ A directed diagram.

A Dynkin quiver $\vec{\Delta}$

$\overset{\text{def}}{\Longleftrightarrow}$ A quiver whose underlying diagram is a simply-laced Dynkin diagram.

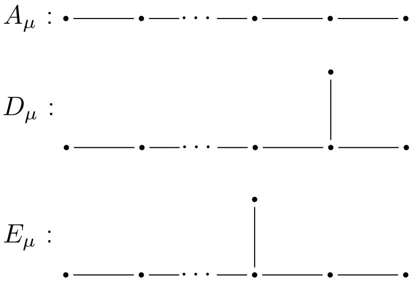

Figure: Simply-laced Dynkin diagrams

Dynkin quivers (diagrams) appear in representation theory, singularity theory, Lie algebra, algebraic geometry, etc.

We consider ADE singularities $\widetilde{f}\colon \mathbb{C}^3 \longrightarrow \mathbb{C}$ defined as follows:

$$\begin{aligned}
A_\mu : \quad & \widetilde{f}(x_1,x_2,x_3) &= \quad & x_1^{\mu+1}+x_2^2+x_3^2 \\
D_\mu : \quad & \widetilde{f}(x_1,x_2,x_3) &= \quad & x_1^{\mu-1}+x_1x_2^2+x_3^2 \\
E_6 : \quad & \widetilde{f}(x_1,x_2,x_3) &= \quad & x_1^4+x_2^3+x_3^2 \\
E_7 : \quad & \widetilde{f}(x_1,x_2,x_3) &= \quad & x_1^3+x_1x_2^3+x_3^2 \\
E_8 : \quad & \widetilde{f}(x_1,x_2,x_3) &= \quad & x_1^5+x_2^3+x_3^2
\end{aligned}$$

$\exists$ equivalent structures between Dynkin quivers and ADE singularities.

e.g.)

- generalized root systems,
- complex manifolds,
- Frobenius manifolds,
- triangulated categories, etc.

We focus on Frobenius manifolds and triangulated categories.

Mirror symmetry
"$\Longleftrightarrow$" an equivalence between algebra and geometry.

There are several ways to formulate mirror symmetry.

A mirror partner of an ADE singularity $\widetilde{f}$ is a pair (f, G_f) consisting of

- $f \in \mathbb{C}[z_1, z_2, z_3]$: an invertible polynomial of chain type,
- G_f : the group of maximal diagonal symmetries of f.

The classical mirror symmetry is an isomorphism of Frobenius manifolds.
The homological mirror symmetry is an equivalence of triangulated categories.

Classical mirror symmetry (CMS):

A Frobenius manifold $M=(M,\eta,\circ,e,E)$

"$\overset{\text{def}}{\Longleftrightarrow}$" a complex manifold M equipped with

- $\eta\colon \mathcal{T}_M\times\mathcal{T}_M\longrightarrow\mathcal{O}_M$: an $\mathcal{O}_M$-bilinear form,
- $\circ\colon \mathcal{T}_M\times\mathcal{T}_M\longrightarrow\mathcal{T}_M$: an $\mathcal{O}_M$-bilinear product,
- $e,E\in\Gamma(M,\mathcal{T}_M)$: two certain vector fields

satisfying some axioms.

$\exists$ 3 ways of constructions of Frobenius manifolds:

(A) $M^A_{(f,G_f)}$: the FJRW theory for (f,G_f) (Fan–Jarvis–Ruan)

(B) $M^B_{(\widetilde{f},\zeta)}$: the theory of primitive forms for $\widetilde{f}$ (K. Saito)

(R) $M^R_{(R(\vec{\Delta}),c)}$: the Weyl group invariant theory for $\vec{\Delta}$ (K. Saito, Saito–Yano–Sekiguchi ,Dubrovin)

$\exists$ isomorphisms of Frobenius manifolds:

$$M^A_{(f,G_f)}\overset{\text{CMS}}{\cong}M^B_{(\widetilde{f},\zeta)}\cong M^R_{(R(\vec{\Delta}),c)}$$

Homological mirror symmetry (HMS):

3 kinds of triangulated categories:

- (A) $\mathrm{HMF}_S^{L_f}(f)$: the homotopy category of matrix factorizations for (f, G_f)
- (B) $\mathcal{D}^b\mathrm{Fuk}^{\rightarrow}(\widetilde{f})$: the derived directed Fukaya category for $\widetilde{f}$
- (R) $\mathcal{D}^b(\vec{\Delta})$: the derived category for $\vec{\Delta}$

$\exists$ equivalences of triangulated categories:

$$\mathrm{HMF}_S^{L_f}(f) \overset{\mathrm{HMS}}{\cong} \mathcal{D}^b\mathrm{Fuk}^{\rightarrow}(\widetilde{f}) \cong \mathcal{D}^b(\vec{\Delta})$$

by P. Seidel, Kajiura–Saito–Takahashi.

Problem (HMS $\Longrightarrow$ CMS ?)

How do we obtain CMS from HMS?

How do we obtain geometric invariants of Frobenius manifolds from triangulated categories?
e.g.) exponents, monodromy data, discriminant, etc...

It is expected that stability conditions on a triangulated category gives an answer (in our setting)!

Remark: There are other approaches to the above Problem.

$\mathcal{D}$: $\mathbb{C}$-linear triangulated category of finite type.

i.e., $\forall\ X, Y \in \mathcal{D}$, $\mathrm{Hom}_{\mathcal{D}}(X, Y)$ is a $\mathbb{C}$-vector space and

$$\dim_{\mathbb{C}} \bigoplus_{p \in \mathbb{Z}} \mathrm{Hom}_{\mathcal{D}}(X, Y[p]) < \infty.$$

Definition 1 (cf. Beilinson–Bernstein–Deligne, Bridgeland).

$\mathcal{A} \subset \mathcal{D}$ is a heart (of a bounded t-structure) in $\mathcal{D}$ if

- $\forall\ E, F \in \mathcal{A}, \quad \mathrm{Hom}_{\mathcal{D}}(E, F[p]) \cong 0$ *for* $p < 0$.
- $\forall\ E \in \mathcal{D},\ E \neq 0, \quad \exists\ k_1 > k_2 > \cdots > k_n$ *: integers*
  $\exists$ *sequence of exact triangles*

$$\begin{array}{ccccccccccc} 0 = F_0 & \longrightarrow & F_1 & \longrightarrow & F_2 & \longrightarrow & \cdots & \longrightarrow & F_{m-1} & \longrightarrow & F_m = E \\ & \nwarrow \swarrow & & \nwarrow \swarrow & & & & & & \nwarrow \swarrow & \\ & A_1 & & A_2 & & & & & & A_m & \end{array}$$

s.t. $A_i \in \mathcal{A}[k_i]$ *for* $i = 1, \ldots, n$.

$\rightsquigarrow$ A heart $\mathcal{A}$ has a structure of an abelian category.
$\rightsquigarrow$ $K_0(\mathcal{A}) \cong K_0(\mathcal{D})$: the Grothendieck group of $\mathcal{A}$ ($\mathbb{Z}$-module).

Definition 2 (Bridgeland).

1. $\mathcal{A}$: *heart in* $\mathcal{D}$.
   A stability function $Z\colon K_0(\mathcal{A}) \longrightarrow \mathbb{C}$ *is a group homomorphism s.t.*

   $$Z(E) \in \mathbb{H}_-, \quad E \in \mathcal{A},\ E \neq 0,$$

   where $\mathbb{H}_- := \{re^{\sqrt{-1}\pi\phi} \in \mathbb{C} \mid R > 0,\ 0 < \phi \leq 1\}$.
2. *A stability condition on* $\mathcal{D}$ *is a pair* $\sigma = (Z, \mathcal{A})$ *consisting of*
   - $\mathcal{A}$: *heart in* $\mathcal{D}$ *and,*
   - $Z\colon K_0(\mathcal{A}) \longrightarrow \mathbb{C}$: *stability function satisfying the "Harder–Narasimhan property" and the "support condition".*

Examples:

1. X : nonsingular projective curve, $\mathcal{D} = \mathcal{D}^b\mathrm{Coh}(X)$
   The slope stability induces a stability condition on $\mathcal{D}$.
2. A : finite dimensional algebra, $\mathcal{D} = \mathcal{D}^b\mathrm{mod}(A)$
   A King's stability induces a stability condition on $\mathcal{D}$.

Fix a stability condition $\sigma = (Z, \mathcal{A})$ on $\mathcal{D}$.

The phase $\phi(E) \in (0,1]$ of $E \in \mathcal{A}$ is $\phi(E) := \dfrac{1}{\pi} \mathrm{Arg}\, Z(E)$.

- $E \in \mathcal{A}$ is σ-(semi)stable of phase $\phi \in (0,1]$
  $\overset{\text{def}}{\iff} \forall\ A \subset E\ (A \neq 0),\quad \phi(A) < \phi(E)$ ($\phi(A) \leq \phi(E)$).
- $E \in \mathcal{D}$ is σ-(semi)stable of phase $\phi \in \mathbb{R}$
  $\overset{\text{def}}{\iff} \exists\ E' \in \mathcal{A}$: σ-(semi)stable of phase $\psi \in (0,1]$, $\exists\ n \in \mathbb{Z}$
  s.t. $E \cong E'[n]$ and $\phi = \psi + n$.

Put

$$\mathrm{Stab}(\mathcal{D}) := \{\text{stability condition on } \mathcal{D}\}.$$

$\exists$ natural topology on $\mathrm{Stab}(\mathcal{D})$.

Theorem 3 (Bridgeland).

The forgetful map

$$\mathcal{Z} \colon \mathrm{Stab}(\mathcal{D}) \longrightarrow \mathrm{Hom}_{\mathbb{Z}}(K_0(\mathcal{D}), \mathbb{C}), \quad (Z, \mathcal{A}) \mapsto Z,$$

is a local homeomorphism.
In particular, $\exists$ complex strutcure on $\mathrm{Stab}(\mathcal{D})$.

Problems

1. What is the complex manifold $\mathrm{Stab}(\mathcal{D}^b(\vec{\Delta}))$?
   $\rightsquigarrow$ It is expected that $\mathrm{Stab}(\mathcal{D}^b(\vec{\Delta}))$ is related to a deformation theory of $\widetilde{f}$.
   $\rightsquigarrow$ $\exists$ Frobenius structure on $\mathrm{Stab}(\mathcal{D}^b(\vec{\Delta}))$?
2. What is a stability condition on $\mathcal{D}^b(\vec{\Delta})$ in terms of $\vec{\Delta}$?

What I studied

In order to construct a Frobenius structure on $\mathrm{Stab}(\mathcal{D})$, I studied stability conditions on $\mathcal{D}^b(\vec{\Delta})$ based on the correspondences between Dynkin quivers $\vec{\Delta}$ and ADE singularities $\widetilde{f}$.

More precisely,

1. I gave a description of $\mathrm{Stab}(\mathcal{D}^b(\vec{\Delta}))$ by full exceptional collections.
2. (Joint work with Atsushi Takahashi)
   We show that the stability function of Kajiura–Saito–Takahashi's stability condition is given by the exponential period associated to a primitive form.

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$\mathcal{D}$: $\mathbb{C}$-linear triangulated category of finite type.

Definition 4.

1. *An object $E \in \mathcal{D}$ is exceptional if*

$$\mathrm{Hom}_{\mathcal{D}}(E, E[p]) \cong \begin{cases} \mathbb{C}, & p = 0, \\ 0, & p \neq 0. \end{cases}$$

2. *An ordered set $\mathcal{E} = (E_1, \ldots, E_\mu)$ of exceptional objects is called exceptional collection if*

$$\mathrm{Hom}_{\mathcal{D}}(E_i, E_j[p]) \cong 0, \quad i > j, \quad \textit{and} \quad p \in \mathbb{Z}.$$

3. *An exceptional collection $\mathcal{E}$ is full if the smallest full triangulated subcategory of $\mathcal{D}$ containing $\mathcal{E}$ is equivalent to $\mathcal{D}$.*

Recall that in the derived category $\mathcal{D}^b(\mathcal{A})$ of an abelian category $\mathcal{A}$,

$$\mathrm{Ext}^p_{\mathcal{A}}(E,F) \cong \mathrm{Hom}_{\mathcal{D}^b(\mathcal{A})}(E, F[p]), \quad p \in \mathbb{Z},\ E, F \in \mathcal{A}.$$

Definition 5 (Macrì).

A full exceptional collection $\mathcal{E} = (E_1, \ldots, E_\mu)$ is Ext if

$$\mathrm{Hom}_{\mathcal{D}}(E_i, E_j[p]) \cong 0 \quad \textit{for} \quad p \le 0.$$

Proposition 6 (Macrì).

$\mathcal{E} = (E_1, \ldots, E_\mu)$ *: full Ext-exceptional collection in* $\mathcal{D}$.
The extension closure $\langle \mathcal{E} \rangle_{\mathrm{ex}} \subset \mathcal{D}$ *is a heart s.t.*

$$\mathrm{Sim}\, \langle \mathcal{E} \rangle_{\mathrm{ex}} = \{E_1, \ldots, E_\mu\}.$$

Corollary 7 (Macrì).

$\mathcal{E} = (E_1, \ldots, E_\mu)$ *: full exceptional collection in* $\mathcal{D}$.
$\exists\ \sigma$ *: stability condition on* $\mathcal{D}$ *s.t.* $E_1, \ldots, E_\mu$ *are* σ*-stable.*

$Q = (Q_0, Q_1)$: acyclic quiver and $Q_0 = \{1, \ldots, \mu\}$.

Question

When is a heart $\mathcal{A}$ in $\mathcal{D}^b(Q)$ given by a full Ext-exceptional collection?

King–Qiu proved that if a heart $\mathcal{A}$ is obtained from $\mathrm{mod}(\mathbb{C}Q)$ by iterated simple tilts, then $\mathcal{A} = \langle \mathrm{Sim}\,\mathcal{A} \rangle_{\mathrm{ex}}$.

Theorem A (O).

Q : acyclic quiver satisfying

(A1) *For* $i, j \in Q_0$, $|\{i \longrightarrow j \in Q_1\}| \leq 1$.

(A2) *For* $i, j, k \in Q_0$, $\exists\, i \longrightarrow j,\ \exists\, j \longrightarrow k \implies \nexists\, i \longrightarrow k$.

$\mathcal{A}$: heart in $\mathcal{D}^b(Q)$.

$\mathcal{A}$ is obtained from $\mathrm{mod}(\mathbb{C}Q)$ by iterated simple tilts.

$\Longrightarrow \exists\, \mathcal{E} = (E_1, \ldots, E_\mu)$ *: full Ext-exceptional collection s.t.*

- $\mathcal{A} = \langle \mathcal{E} \rangle_{\mathrm{ex}}$,
- $\mathrm{Sim}\,\mathcal{A} = \{E_1, \ldots, E_\mu\}$.

Definition 8 (Dimitrov–Katzarkov).

σ : stability condition on $\mathcal{D}$,
$\mathcal{E} = (E_1, \ldots, E_\mu)$: exceptional collection in $\mathcal{D}$.
$\mathcal{E}$ is σ-exceptional collection if

- *$E_1, \ldots, E_\mu$ are σ-stable,*
- *$\mathcal{E}$ is Ext, and*
- *$\exists\, r \in \mathbb{R}$ s.t. $r < \phi(E_i) \leq r + 1$ for $i = 1, \ldots, \mu$.*

Macrì and Dimitrov–Katzarkov showed that, for the ℓ-Kronecker quiver and the affine $A_{1,2}^{(1)}$ quiver, any stability condition σ on $\mathcal{D}^b(Q)$ has a full σ-exceptional collection, respectively.

Theorem B (O).

$\vec{\Delta}$: Dynkin quiver.

For each stability condition σ on $\mathcal{D}^b(\vec{\Delta})$, $\exists$ full σ-exceptional collection.

$$\mathrm{FEC}(\vec{\Delta}) := \left\{ \mathcal{E} = (E_1, \dots, E_\mu) \;\middle|\; \begin{array}{c} \mathcal{E} : \text{ full exc coll in } \mathcal{D}^b(\vec{\Delta}) \\ E_i \in \mathrm{mod}(\mathbb{C}\vec{\Delta}) \end{array} \right\} \Big/ \cong .$$

U. Seidel proved

$$|\mathrm{FEC}(\vec{\Delta})| = \frac{\mu!\, h^\mu}{|W|} < \infty,$$

where
- h : the Coxeter number of $\vec{\Delta}$,
- W : the Weyl group for $\vec{\Delta}$.

For $\mathcal{E} = (E_1, \dots, E_\mu) \in \mathrm{FEC}(\vec{\Delta})$, define an open subset $\Theta_\mathcal{E}$ by

$$\Theta_\mathcal{E} := \{\sigma \in \mathrm{Stab}(\mathcal{D}^b(\vec{\Delta})) \mid E_1, \dots, E_\mu \text{ are } \sigma\text{-stable}\} \subset \mathrm{Stab}(\mathcal{D}^b(\vec{\Delta})).$$

Theorem C (O).

The set $\{\Theta_\mathcal{E}\}_{\mathcal{E} \in \mathrm{FEC}(\vec{\Delta})}$ *is a finite open covering :*

$$\mathrm{Stab}(\mathcal{D}^b(\vec{\Delta})) = \bigcup_{\mathcal{E} \in \mathrm{FEC}(\vec{\Delta})} \Theta_\mathcal{E}.$$

$\widetilde{f}$: ADE singularity of type corresponding to $\vec{\Delta}$.

$\mathcal{B}(\widetilde{f}) :=$ the set of distinguished basis of vanishing cycles for $\widetilde{f}$.

The group $\mathbb{Z}_2^\mu$ acts on $\mathcal{B}(\widetilde{f})$ as the change of signs.

(Gusein-Zade, Crawley-Boevey) $\exists$ bijection

$$\mathcal{B}(\widetilde{f})/\mathbb{Z}_2^\mu \xrightarrow{1:1} \mathrm{FEC}(\vec{\Delta}), \quad \mathcal{L} \mapsto \mathcal{E}_{\mathcal{L}}$$

Hence,

$$\mathrm{Stab}(\mathcal{D}^b(\vec{\Delta})) = \bigcup_{\mathcal{L}\in\mathcal{B}(\widetilde{f})/\mathbb{Z}_2^\mu} \Theta_{\mathcal{E}_{\mathcal{L}}}.$$

To understand this equality (or a Frobenius structure on $\mathrm{Stab}(\mathcal{D}^b(\vec{\Delta}))$) more precisely, we need to study what is a stability function $Z\colon K_0(\mathcal{A}) \longrightarrow \mathbb{C}$ for each stability condition $\sigma = (Z, \mathcal{A})$.

- A polynomial $f \in \mathbb{C}[z_1, \dots, z_n]$ is an invertible polynomial of chain type if

$$f(z_1, \dots, z_n) := z_1^{a_1} z_2 + \cdots + z_{n-1}^{a_{n-1}} z_n + z_n^{a_n}, \quad a_i \in \mathbb{Z}_{\geq 2}.$$

  Define a $\mathbb{C}$-vector space Ω_f by

$$\Omega_f := \Omega^n(\mathbb{C}^n)/df \wedge \Omega^{n-1}(\mathbb{C}^n).$$

  $\exists$ $\mathbb{Q}$-grading on Ω_f given by fractional weights of f.

- The group of maximal diagonal symmetries G_f of f is defined as

$$G_f := \{(\lambda_1, \dots, \lambda_n) \in (\mathbb{C}^*)^n \mid f(\lambda_1 z_1, \dots, \lambda_n z_n) = f(z_1, \dots, z_n)\} .$$

Define a $\mathbb{Q}$-graded $\mathbb{C}$-vector space Ω_{f,G_f} by

$$\Omega_{f,G_f} := \bigoplus_{g \in G_f} \Omega_{f,g}, \quad \Omega_{f,g} := (\Omega_{f^g})^{G_f}(-\mathrm{age}(g)),$$

where for $g \in G_f$,

- $f^g := f|_{\{z \in \mathbb{C}^\mu \mid g \cdot z = z\}}$,
- $\mathrm{age}(g) \in \mathbb{Q}$ is the age of g.

The $\mathbb{C}$-vector space Ω_{f,G_f} for (f, G_f) is an analogue of the total Hodge cohomology $H^*(X;\mathbb{C})$ for an algebraic variety X.

$\exists\ \mathbb{S}_{f,G_f} : \Omega_{f,G_f} \times \Omega_{f,G_f} \longrightarrow \mathbb{C}$: $\mathbb{C}$-bilinear form
s.t. $\mathbb{S}_{f,G_f}$ on Ω_{f,G_f} is an analogue of the polarization on $H^*(X;\mathbb{C})$.

For an invertible polynomial of chain type f, define $\widetilde{f}$ by

$$\widetilde{f}(x_1, \ldots, x_n) \coloneqq x_1^{a_1} + x_1 x_2^{a_2} + \cdots + x_{n-1} x_n^{a_n}.$$

Kreuzer showed $\exists$ isomorphism of $\mathbb{Q}$-graded $\mathbb{C}$-vector spaces

$$\mathbf{mir} : \Omega_{f,G_f} \xrightarrow{\cong} \Omega_{\widetilde{f}}.$$

Proposition 9 (O–Takahashi).

$$\mathbf{mir} : (\Omega_{f,G_f}, \mathbb{S}_{f,G_f}) \xrightarrow{\cong} (\Omega_{\widetilde{f}}, \mathbb{S}_{\widetilde{f}})$$

Theorem 10 (O–Takahashi, Aramaki–Takahashi).

$\exists$ $\mathrm{Ch}_\Gamma : K_0(\mathrm{HMF}_S^{L_f}(f)) \longrightarrow \Omega_{f,G_f}$: group homomorphism s.t. the following diagram commutes:

$$\begin{array}{ccc} (K_0(\mathrm{HMF}_S^{L_f}(f)), \chi) & \xrightarrow{\cong \text{ due to AT}} & (H_n(\mathbb{C}^n, \mathrm{Re}(\widetilde{f}) \gg 0; \mathbb{Z}), \mathbb{S}) \\ \Big\downarrow \mathrm{Ch}_\Gamma & & \Big\downarrow \mathbb{D} \\ (\Omega_{f,G_f}, \mathbb{S}_{f,G_f}) & \xrightarrow{\quad \mathbf{mir} \quad} & (\Omega_{\widetilde{f}}, \mathbb{S}_{\widetilde{f}}) \end{array},$$

where

- $\mathbb{D}$ *: the "Poincaré duality map",*
- χ *: the Euler form on $K_0(\mathrm{HMF}_S^{L_f}(f))$,*
- $\mathbb{S}$ *: the "Seifert form" on $H_n(\mathbb{C}^n, \mathrm{Re}(\widetilde{f}) \gg 0; \mathbb{Z})$.*

Moreover, $\mathbf{mir} : \mathrm{Im}\,\mathrm{Ch}_\Gamma \cong \mathrm{Im}\,\mathbb{D}$ is equivariant w.r.t. a cyclic group action.

For $\Gamma \in H_3(\mathbb{C}^3, \mathrm{Re}(\widetilde{f}) \gg 0; \mathbb{Z})$, the exponential period mapping $\displaystyle\int_\Gamma e^{-\widetilde{f}}\zeta$ can be calculated as the Gamma function and fractional weights of (f, G_f).

Theorem D (O–Takahashi, Kajiura–Saito–Takahashi).

$\vec{\Delta}$: Dynkin quiver,
$\widetilde{f}\colon \mathbb{C}^3 \longrightarrow \mathbb{C}$: ADE singularity.

$\exists\ \sigma_0 = (Z, \mathcal{A}) \in \mathrm{Stab}(\mathcal{D}^b(\vec{\Delta}))$ *s.t.*

1. *(KST)* $\mathcal{A} \cong \mathrm{mod}(\mathbb{C}\vec{\Delta}_{\mathrm{principal}})$ *and,*
2. *(OT)* $Z\colon K_0(\mathcal{D}^b(\vec{\Delta})) \cong H_3(\mathbb{C}^3, \mathrm{Re}(\widetilde{f}) \gg 0; \mathbb{Z}) \longrightarrow \mathbb{C}$ *is*

$$Z(E) = \int_{\psi_0(E)} e^{-\widetilde{f}}\zeta,$$

where

- $\psi_0\colon K_0(\mathcal{D}^b(\vec{\Delta})) \xrightarrow{\cong} H_3(\mathbb{C}^3, \mathrm{Re}(\widetilde{f}) \gg 0; \mathbb{Z})$ *is an isomorphism,*
- $\zeta = [dx_1 \wedge dx_2 \wedge dx_3]$ *is a primitive form.*

Thank you very much !